EE 362: Signals and Systems Name: Solutions

Spring 2019

Homework #8 (Due: Thursday, March 28, 2019)

- Circle or box your answer (don't forget units).
- **Show** your steps.
- Use 8.5" x 11" paper, and trim off any ragged edges.
- **Staple** your work together and write your name neatly.

Work out the following problems from your textbook (Edward W. Kamen and Bonnie S. Heck, Fundamentals of Signals and Systems Using the Web and MATLAB, 3rd Ed, Prentice Hall, 2006)

1. Work out problem 4.1, part (a) [3 points]

2. Work out problem 4.4, part (a). [3 points]

3. Work out problem 4.5, parts (a), (b), (c), and (e). [4 points; 1 point for each part]

4. Work out problem 4.6, parts (b) and (c). [6 points; 3 points for each part]

5. Work out problem 4.9, parts (a) and (c). [6 points; 3 points for each part]

You can use MATLAB to solve the above problems. However, to receive credits for questions solved using MATLAB, you must printout your MATLAB script file and the results; hand writing/sketch is not acceptable for MATLAB solutions.

4.1-a

$$x[n] = 2P_4[n-2] - 2P_2[n-6]$$

From table 4.1, $P[n] \leftrightarrow \frac{\sin[(q+\frac{1}{2})\Omega]}{\sin(\Omega/2)}$

where q is one half of pulse time ($q = L/2$)

i.e., $P_{2q}[n] \leftrightarrow \frac{\sin[(q+\frac{1}{2})\Omega]}{\sin(\Omega/2)}$

$$\therefore 2P_4[n] \leftrightarrow 2\frac{\sin[(2+\frac{1}{2})\Omega]}{\sin[\Omega/2]}$$

$$-2P_2[n] \leftrightarrow -2\frac{\sin[(1+\frac{1}{2})\Omega]}{\sin[\Omega/2]}$$

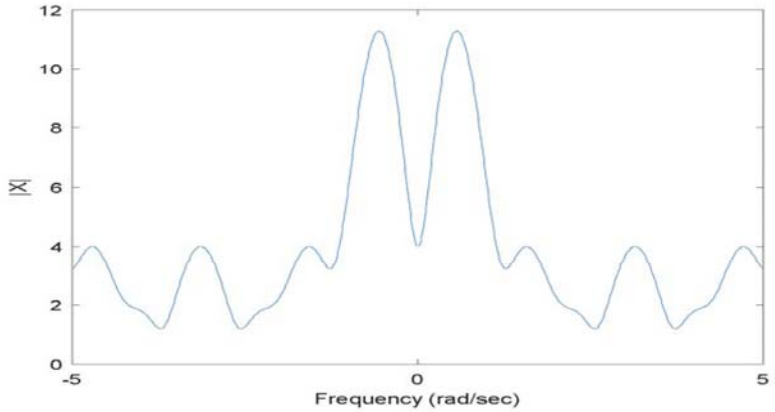
From table 4.2, $x[n-q] \leftrightarrow X(\Omega)\, e^{-jq\Omega}$

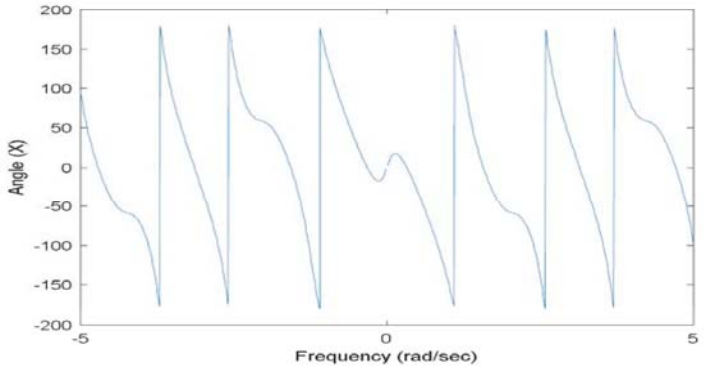
$$\therefore X(\Omega) = 2\frac{\sin[(2+\frac{1}{2})\Omega]}{\sin(\Omega/2)} e^{-j2\Omega} - 2\frac{\sin[(1+\frac{1}{2})\Omega]}{\sin(\Omega/2)} e^{-j6\Omega}$$

$$= 2\left[\frac{\sin(2.5\Omega)}{\sin(\Omega/2)} e^{-j2\Omega} - \frac{\sin(1.5\Omega)}{\sin(\Omega/2)} e^{-j6\Omega}\right]$$

Plots of problem 4.1 a

```
w=-5:0.01:5;
X=2*((sin(2.5*w)./sin(w/2)).*exp(-j*2*w)-(sin(1.5*w)./sin(w/2)).*exp(-
j*6*w));
plot(w,abs(X));
xlabel('Frequency (rad/sec)')
ylabel('|X|')
figure
plot(w,angle(X)*180/pi);
xlabel('Frequency (rad/sec)')
ylabel('Angle (X)')
```





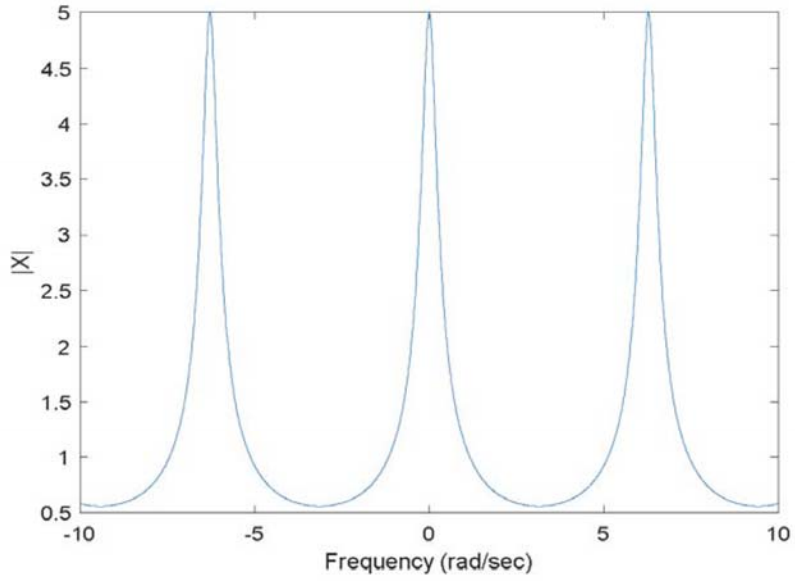
4.4-a $x[n] = (0.8)^n u[n]$

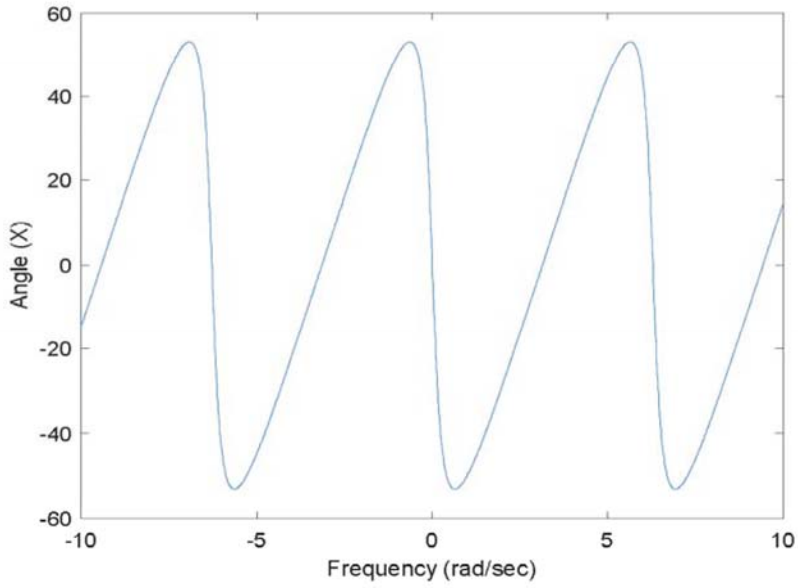
From table 4.1, $a^n u[n] \longleftrightarrow \frac{1}{1-a e^{-j\Omega}}$, $|a|<1$

$\therefore X[\Omega] = \frac{1}{1-0.8e^{-j\Omega}} = \frac{e^{j\Omega}}{e^{j\Omega}-0.8}$

Plots of problem 4.4 a

```
w=-5:0.01:5;
X=1./(1-0.8*exp(-j*w));
plot(w,abs(X));
xlabel('Frequency (rad/sec)')
ylabel('|X|')
figure
plot(w,angle(X)*180/pi);
xlabel('Frequency (rad/sec)')
ylabel('Angle (X)')
```





4.5

$$X(\Omega) = \frac{1}{e^{j\Omega} + b}$$

a

$V[n] = x[n-5]$ ← time shift property

From table 4.2, $x[n-q] \leftrightarrow X(\Omega)\, e^{-jq\Omega}$

$\therefore V(\Omega) = \frac{1}{e^{j\Omega} + b}\, e^{-j5\Omega}$

b $V[n] = x[-n]$ ← time reversal property

From table 4.1, $x[-n] \leftrightarrow X(-\Omega) = \overline{X(\Omega)}$

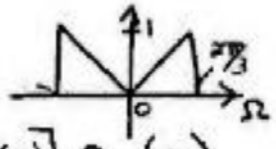
$\therefore V(\Omega) = \left(\overline{\frac{1}{e^{j\Omega} + b}}\right) = \frac{1}{e^{-j\Omega} + b}$

c $V[n] = n\,x[n]$ ← multiplication by n

$$V(\Omega) = j\frac{d}{d\Omega}X(\Omega) = j\,\frac{0 - je^{j\Omega}}{(e^{j\Omega}+b)^2} = \frac{e^{j\Omega}}{(e^{j\Omega}+b)^2}$$

(e) $V(\Omega) = \frac{1}{(e^{j2\Omega}+b)^2}$

4.6

(b) $x[n] = 2v[n]\cos(\pi n)$ where $V(\Omega)P_{2\pi}(\Omega) =$ 

Now $V(\Omega)P_{\pi}(\Omega) =$ [1 - freq. function in part (a)] $P_{2\pi}(\Omega)$

so $v[n] = \delta[n] - \frac{1}{3}\,\mathrm{sinc}^2\left(\frac{n}{3}\right)$

and $x[n] = 2\left[\delta[n] - \frac{1}{3}\,\mathrm{sinc}^2\left(\frac{n}{3}\right)\right]\cos(\pi n)$

$x[n] = 2\delta[n] - \frac{2}{3}\,\mathrm{sinc}^2\left(\frac{n}{3}\right)\cos(\pi n)$

(c) $X(\Omega)P_{\pi}(\Omega) = P_{\frac{\pi}{3}}(\Omega) + P_{\frac{2\pi}{3}}(\Omega)$

so $x[n] = \frac{1}{3}\left[2\,\mathrm{sinc}\frac{2n}{3} + \mathrm{sinc}\frac{n}{3}\right]$

Explanation of 4.6.b (Also, you can follow the steps provided in review session material)

4.6 b

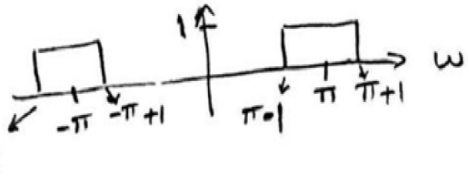
From table 4.2,

$v[n]\cos\Omega_0 n \longleftrightarrow \frac{1}{2}\left[V(\Omega+\Omega_0) + V(\Omega-\Omega_0)\right]$

$\therefore\ v[n]\cos(n\pi) \longleftrightarrow \frac{1}{2}\left[V(\Omega+\pi) + V(\Omega-\pi)\right]$

$\therefore\ 2v[n]\cos(n\pi) \longleftrightarrow \left[V(\Omega+\pi) + V(\Omega-\pi)\right]$

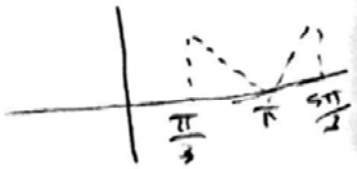
Now, $\cos(n\pi)$ takes care of the frequency shift and frequency reversal. In other words, if $2V(\Omega)$ is 

$\therefore\ 2v[n]\cos(n\pi) \longleftrightarrow$ 

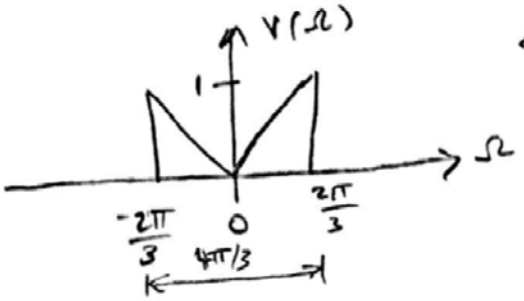
$\therefore\ x[n] = 2v[n]\cos(n\pi)$

Also, convolution of $P_\tau(t) * P_\tau(t) = \tau\left(1 - \frac{2|t|}{2\tau}\right) P_{2\tau}(t)$ ← verify

$\therefore P_{\tau/2}(t) * P_{\tau/2}(t) = \frac{\tau}{2}\left(1 - \frac{2|t|}{\tau}\right) P_\tau(t)$

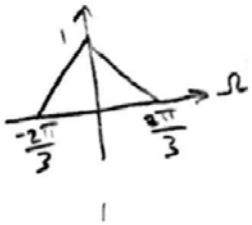


in our example



$\therefore \frac{2}{\tau} P_{\tau/2}(t) * P_{\tau/2}(t) = \left(1 - \frac{2|t|}{\tau}\right) P_\tau(t)$

$\therefore \frac{2}{(4\pi/3)} P_{\frac{4\pi/3}{2}}(\Omega) * P_{\frac{4\pi/3}{2}}(\Omega) = \underbrace{\left(1 - \frac{2|\Omega|}{4\pi/3}\right) P_{\frac{4\pi}{3}}(\Omega)}$ =



$\frac{3}{2\pi} P_{\frac{2\pi}{3}}(\Omega) * P_{\frac{2\pi}{3}}(\Omega) = \left(1 - \frac{2|\Omega|}{4\pi/3}\right) P_{\frac{4\pi}{3}}(\Omega)$

$\therefore V(\Omega) = 1 - \frac{3}{2\pi}\left(P_{\frac{2\pi}{3}}(\Omega) * P_{\frac{2\pi}{3}}(\Omega)\right)$

$$V[n] = \delta[n] - \frac{3}{2\pi}(2\pi)\left[\left(\frac{2\pi/3}{2\pi}\,\text{sinc}\left(\frac{n \times 2\pi/3}{2\pi}\right)\right)\left(\frac{2\pi/3}{2\pi}\,\text{sinc}\left(\frac{n \times 2\pi/3}{2\pi}\right)\right)\right]$$

$$= \delta[n] - 3\left(\frac{1}{3}\,\text{sinc}\left(\frac{n}{3}\right)\right)^2$$

$$\therefore V[n] = \delta[n] - \frac{1}{3}\,\text{sinc}^2\left(\frac{n}{3}\right)$$

$\because x[n] = 2V[n]\cos(n\pi)$

$$\therefore x[n] = 2\left[\delta[n] - \frac{1}{3}\,\text{sinc}^2\left(\frac{n}{3}\right)\right]\cos(n\pi)$$

$$= 2\delta[n] - \frac{2}{3}\,\text{sinc}^2\left(\frac{n}{3}\right)\cos(n\pi)$$

4.9(a) $X_n = 1 + e^{-j\pi n} = 1 + \cos \pi n$, $n = 0, 1, 2, 3$

$X_0 = 2, \quad X_1 = 0, \quad X_2 = 2, \quad X_3 = 0$

(c) $X_n = 1 + e^{-j\pi n/2} - e^{-j\pi n} - e^{-j3\pi n/2}$, $n = 0, 1, 2, 3$

$R_n = 1 + \cos \frac{\pi n}{2} - \cos n\pi - \cos \frac{3\pi n}{2}$

$R_0 = 0, \quad R_1 = 2, \quad R_2 = 0, \quad R_3 = 2$

$I_n = -\sin \frac{n\pi}{2} + \sin \pi n + \sin \frac{3\pi n}{2}$

$I_0 = 0, \quad I_1 = -2, \quad I_2 = 0, \quad I_3 = 2$

Thus $X_0 = 0, \quad X_1 = 2 - j2, \quad X_2 = 0, \quad X_3 = 2 + j2$